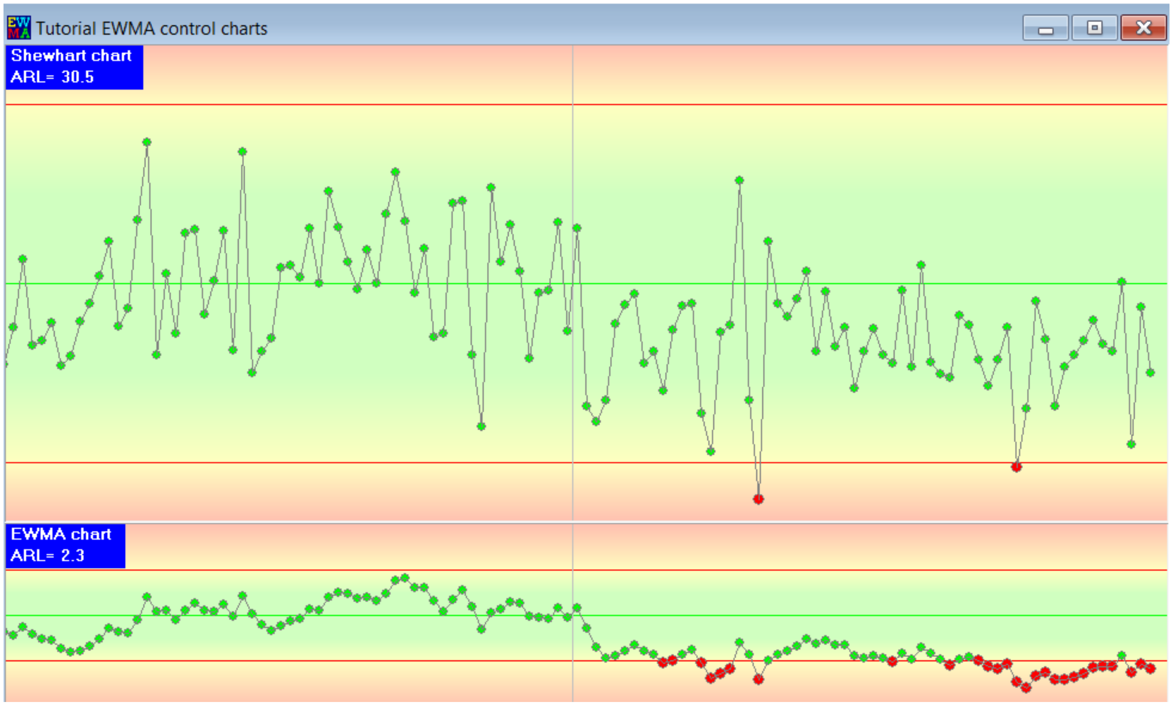
SPC

LESSON: Time-Weighted Charts

Quality Methods

Time Weighted Charts



Dr. Diane Evans

Other VARIABLE Control Charts: Time-Weighted Charts: MA, EWMA, CUSUM

Deming, 1993: "The Shewhart charts ($\bar{X}$, R, I-MR, etc.) do a **good job under a wide range of conditions**. No one has yet wrought improvement."

Ryan, 2000: "Times change, however, and charts with superior properties have been developed. This is to be expected. How many people would want to drive a car that was made in 1924, the year that Shewhart sketched out the idea for a control chart?"

(continued): "This doesn't necessarily mean the modern procedures should replace Shewhart charts, but modern methods should be studied and given serious **consideration** for use in **applications** for which they are ***well-suited***."

Moving Average (MA) Charts & Exponentially Weighted Moving Average (EWMA) Charts: Charts for detecting *small shifts* in the *process mean*.

- These charts **"accumulate" data** over time; i.e., use past, current and future data.
  - ◊ Unlike $\bar{X}$-R and I-MR charts, **all** the data collected over time may be used to determine the "control" status of a process.
  - ◊ If we start collecting accumulated data directly after a **process shift**, then the **accumulated data** will reveal **when a process mean has shifted**.
  - ◊ If there has been a **shift in the mean**, we want a **smaller Average Run Length (ARL)** than a Shewhart chart would provide.

- Developed by S.W. Roberts in 1959
- The **EWMA methodology** is ***not sensitive*** to **normality assumptions**.
- The EWMA is often superior to the CUSUM charting technique for **detecting "larger" shifts**.
- **READ the handout on MA and EWMA charts** from the Mitra text at the end of these lecture notes.
- **ASSUMPTION:** The samples obtained over time must be **independent**. If that assumption is violated, there are two possible scenarios:
-

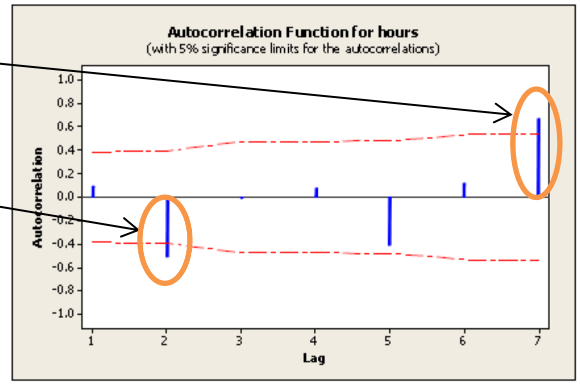
a. Control limits that are too narrow: Positive autocorrelation (e.g., low values tend to be followed by other low values, or high values tend to follow other high values) can increase the frequency of false alarms.

b. Control limits that are overly wide: Negative autocorrelation (e.g., processes that frequently overcorrect – e.g., Deming's Funnel Example) may lead to missing special causes of variation.

Check autocorrelation in **Minitab: Stat > Time Series > Autocorrelation**

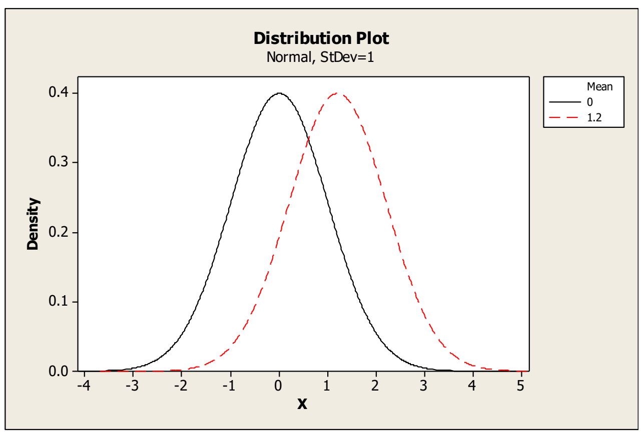
Positive autocorrelation at lag 7; positive dependency in every 7th point; e.g, Monday affects Monday, Tuesday affects Tuesday, ...

Negative autocorrelation at lag 2; every 2nd point flip-flops: high, low, high, low, ...

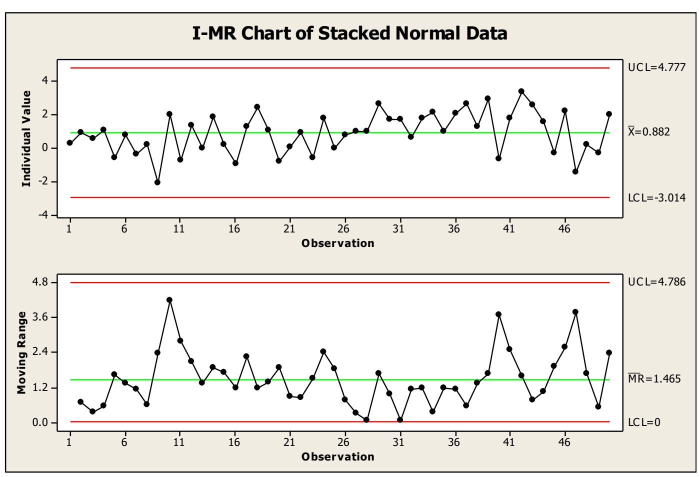


Significant dependency is at lags beyond the red "significance" lines

Example 1. In Minitab, I generated a column of 25 observations from a Normal (0, 1) distribution and then generated another column of 25 observations from a Normal (1.2, 1) distribution (where the new mean has shifted 1.2 standard deviations to the right of the previous distribution's mean). I stacked the two columns of data into a new column.
The first 25 observations are Normal (0, 1) and the next 25 observations are Normal (1.2, 1).

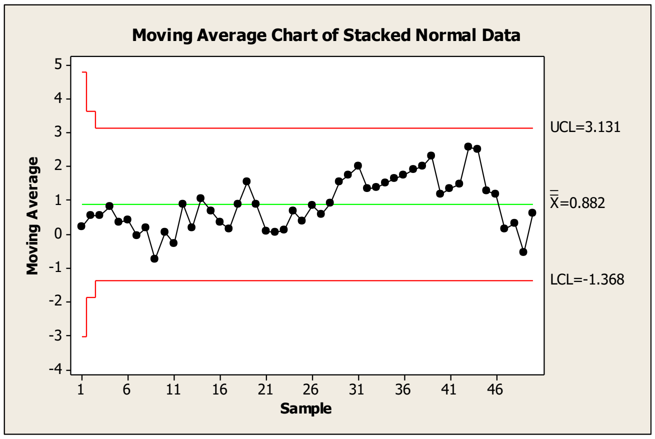


(a) Construct an ***I-MR* chart** for the 50 stacked data points. Are there any out of control points noted on the individuals chart (using only Rule 1)? Can you detect the shift in the data (around point 25) from this chart?

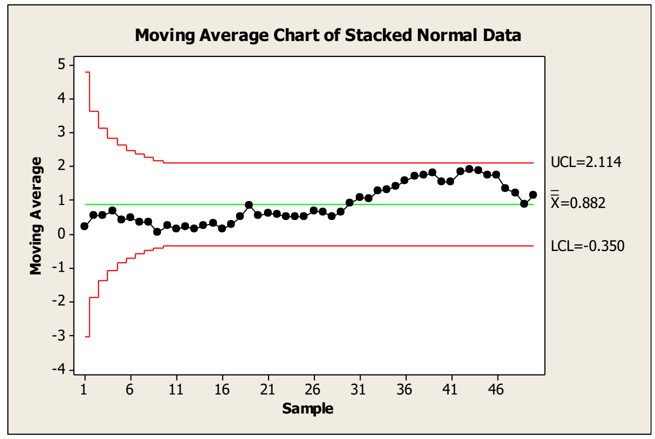


(b) Next construct a **Moving Average chart** for a **span of *w* = 3** for the 50 stacked data points.

Stat > **Control Charts** > **Time-Weighted Charts** > **Moving Average;** Subgroup sizes: 1; Length of MA: 3, 10. Can you detect a shift of the mean in the data from this chart?

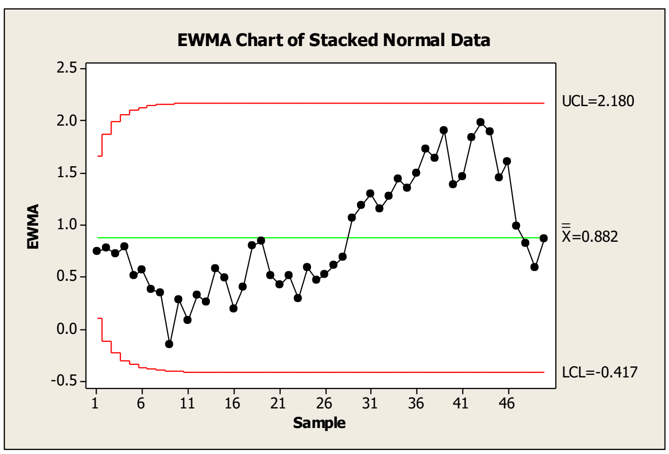


Span $w = 3$

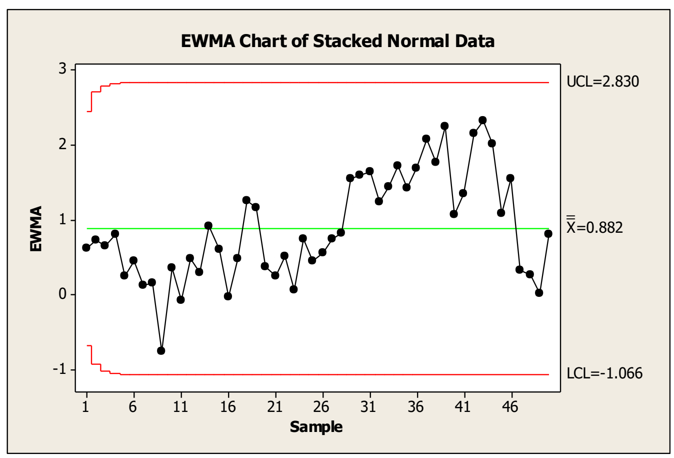


Span $w = 10$

(c) Next construct an Exponentially Weighted Moving Average chart for the 50 stacked data points. **Stat** > **Control Charts** > **Time-Weighted Charts** > **EWMA;** Subgroup sizes: 1; Weight of EWMA: 0.2, 0.4. Can you detect a shift of the mean in the data from this chart?

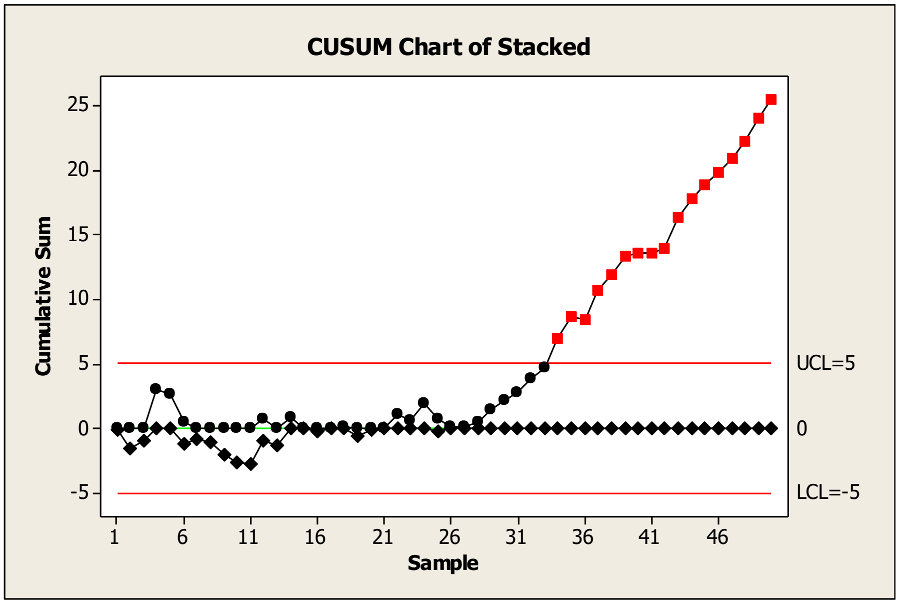


Weight $r = 0.2$



Weight $r = 0.4$

(d) Last construct a CUSUM chart using the one-sided UCL and LCL. Since the standard deviation of the process is 1, then *h* must be 5, since h = 5σ. So, the UCL and LCL must be 5 and -5, respectively. Also, we want to detect a shift in the mean of 1, so *k* is ½ = 0.5.



QUICK overview of CUSUM chart:

- A **CUSUM** chart tracks the **sum of the differences between sample values and the target** (see golf example below)
- A **CUSUM** chart is used for **“in-control” processes to detect small shifts** away from the target
  - ◊ **More efficient than an $\bar{X}$ chart when process off-target** by $0.5\sigma/\sqrt{n}$ to $2\sigma/\sqrt{n}$
  - ◊ The **point of change in process mean is easily located**, which is useful when trying to determine when the change occurred and the cause of the change
  - ◊ **Greater visual impact** then $\bar{X}$-R chart
  - ◊ **Efficient** for **sample size of 1**
- Suitable for processes in which it takes some time to produce a single item
- Though CUSUM charts have been well researched and developed, many quality control practitioners do not use them
  - ◊ Their lack of use may be due to a *lack of instruction* on CUSUM charts in many classes or the workplace
- **Disadvantages**
  - ◊ Sensitive to the normality assumption
  - ◊ More complex to use & more complex to understand

History of CUSUM charts

- These charts were proposed and developed in the **1950s** and **60s** as an alternative to the $\bar{X}$ chart
- They were developed because $\bar{X}$ **charts** are **insensitive to small or gradual changes in the process mean**
- **They are more sensitive than** $\bar{X}$ charts because they contain **information from all samples** that have been taken rather than just the most recent sample

Using the game of golf to explain CUSUM charts:

- In essence, for each hole in a round of golf, there are a specified number of times in which one should strike the ball, until it eventually drops into the hole.

  Example. On a **par 4 hole**, if you get the ball in the hole in **4 shots**, then you held **par**. If you were able to do this with only **three shots** (a "birdie") then you are **"1 under par"** hence your ***cumulative sum is -1***. This is continued throughout the course with the **ultimate winner** having the **lowest CUSUM** score.
- Imagine a golfer who is holding par for the first 13 holes, then suddenly has five successive birdies towards the end of the round. The final CUSUM is therefore -5 and from viewing a CUSUM chart it would be clear to see when the "process" shifted.

Minitab generates two kinds of CUSUMs. We'll use the first. The 2nd is very difficult formula-wise to understand.

- Two one-sided CUSUMs. The **upper CUSUM detects upward shifts in process** level and the **lower CUSUM detects downward shifts**. This chart uses upper and lower control limits to determine when the process is out of control.
- One two-sided CUSUM. This chart uses a ***V-mask***, rather than control limits, to determine when an out-of-control situation has occurred.

Example 2. Suppose you work at a carnival and run the merry-go-round or carousel of horses. The horses move up and down a certain distance from an ideal baseline position. The file **Lesson21DATA_CUSUM_Carousel** is the distance (in inches) from the actual (A) position of a point on a given horse to the baseline (B) position.

To ensure that this given horse is "finishing" in the correct spot (0 distance from the baseline), you took:

- Five measurements each working day, from September 28 through October 15, and
- Two sets of five measurements per day (morning/afternoon) from the 18th through the 25th on the finishing spot (A) on the horse to the baseline position (B).

First, we'll create $\bar{X}$-R charts in Minitab of the distance from the finishing spot (A) to the baseline position (B).

- The subgroup data is all in one column (C1) and the subgroup sizes are in the "Days" (C3) column

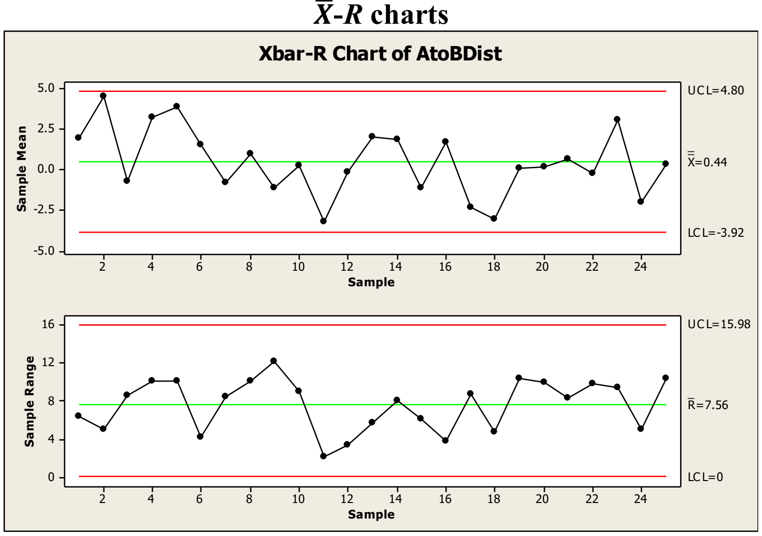
AtoBDist	Month	Day
-0.44025	9	28
5.90038	9	28
2.08965	9	28
0.09998	9	28
2.01594	9	28
4.83012	9	29
3.78732	9	29 …

Select "All observations for a subgroup are in one column" (C1)
Subgroups sizes: "Days" (C3)

- I'm using $\bar{R}$ to estimate the process standard deviation $\hat{\sigma}$
- I'm going to run **3 tests for determining out of control points:**

I'll run the usual "beyond LCL or UCL" test. To notice a shifting mean, I'll use: "9 points in a row on same side of center line," "6 points in a row all increasing or all decreasing"

$\bar{X}$-R charts



CUSUM

- For the "Carousel" example, none of the subgroups were deemed out of control using these 3 rules on the $\bar{X}$ and R charts
- We can look for small shifts away from the target by plotting the CUSUM chart

Rules and Procedures

- When we have data in subgroups, the **mean of the observations in each subgroup is calculated**, CUSUM statistics are then formed from these means
- All subgroups must be the same size
- If a subgroup size is greater than size 1, then the process standard deviation, $\hat{\sigma}$, is estimated using the pooled standard deviation (by default in Minitab). You can have Minitab estimate σ by using the $\bar{R}$, or you can also enter a historical value for σ.

The V-mask

The CUSUM chart gives rise to an out-of-control signal whenever the slope (from point to point) becomes too great – in either a positive or negative direction.

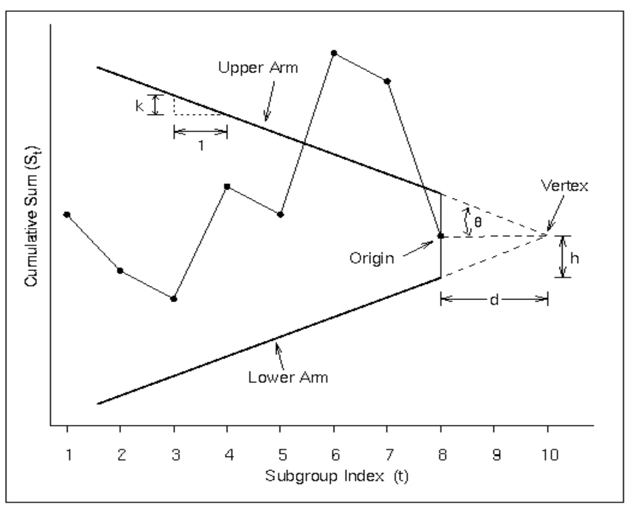
The mask is placed on the control chart in the following manner:

- The axis of the mask is horizontal with the truncated V pointing in the direction of increasing sample number.
- The center of the truncated vertex is placed on the latest value of the CUSUM, S_m.
- If the CUSUM graph goes beyond either edge of the V-mask, the process is deemed out of control.
- A V-mask is defined by two parameters, h and k, which define the width of the “nose” of the V-mask and the slope of the edges.
- The sensitivity of the chart depends on the value of these. For the standard V-mask, *h* = 5 and *k* = 0.5.

“WE STRONGLY ADVISE THE QUALITY ENGINEER NOT TO USE THE V-MASK PROCEDURE.”
Montgomery, Introduction to Statistical Quality Control, 1991:

“Perhaps the biggest problems with the V-mask is the ambiguity associated with Type I Error α and Type II Error β.”

To construct a ***standard V-mask***, the “nose” is *h* standard errors, $h \cdot \frac{\sigma}{\sqrt{n}}$, wide and the slope of the two “arms” are *k* standard error $k \cdot \frac{\sigma}{\sqrt{n}}$ and -*k* standard error.



How are *h* and *k* determined?

First, the amount of Type I Error α and Type II Error β that you will allow must be chosen.

Let $\delta = \frac{\Delta \overline{X}}{\sigma_{\overline{X}}}$, where $\Delta \overline{X}$ is the shift in the process mean we want to be able to detect.

Then $h = |\delta|^{-1} \log((1-\beta)/(\alpha/2))$ and $k = |\delta|/2$.

The Tabular or Algorithmic CUSUM

The tabular CUSUM works by calculating two statistics:

- C^+: ***the one-sided upper Cusum***, which is equal to the sum of all deviations above the target, and
- C^-: ***the one-sided lower Cusum***, which is equal to the sum of all deviations below the target

  Both are plotted on the same graph with time (trial number) as the horizontal axis. This is called the ***CUSUM status chart.***

C^+ and C^- are the accumulation of deviations from the target, *T*, that are greater than a specified value, *k*, and C^+ is reset to zero upon becoming negative and C^- is reset to positive upon becoming positive.

For single observations (samples of size 1), we have:

$C_i^- = \min\{0, x_i - (T - k) + C_{i-1}^-\}$,
$C_i^+ = \max\{0, x_i - (T + k) + C_{i-1}^+\}$

Minitab uses the following to determine C_i^+ and C_i^- for samples of size *n*:

$$C_i^- = \min\{0, \bar{x}_i - \left(T - \frac{k\sigma}{\sqrt{n}}\right) + C_{i-1}^-\}$$

$$C_i^+ = \max\{0, \bar{x}_i - \left(T + \frac{k\sigma}{\sqrt{n}}\right) + C_{i-1}^+\}$$

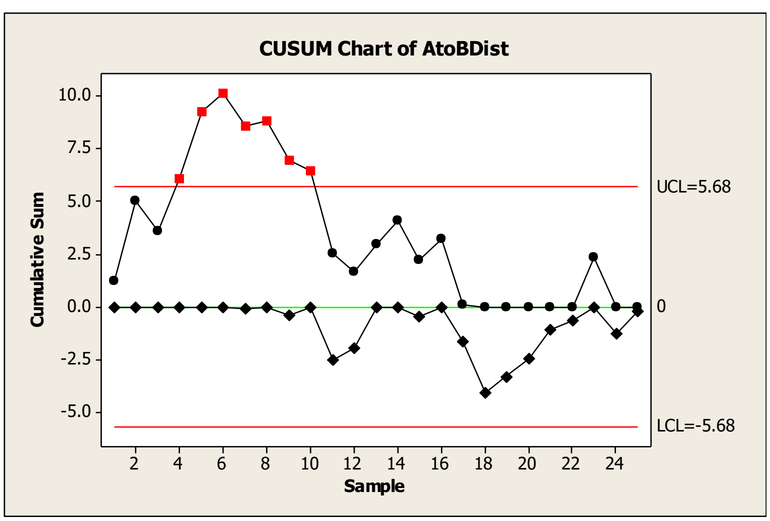
where the starting values are $C_0^+ = 0$ and $C_0^- = 0$. The value *k*, is called the ***reference value*** (or ***allowance or slack value***) and is often chosen to be halfway between the target value *T* and an out-of-control value of the shifted mean μ_{SH} that is to be detected quickly. That is $k = (\delta/2).\delta$ **where δ is the shift in the mean that we wish to detect.**

If C^+ becomes negative, it is reset to 0. If C^- becomes positive, it is reset to 0.

If either C^+ or C^- exceeds (in absolute value) a value called the ***decision interval, h***, the process is deemed out of control. **A standard choice for h is five times the process standard deviation**.

For *T* = 0, *h* = 4, *k* = 0.5, we obtain the following graph for the CUSUM chart with tabular style for the Carousel

example:



Example 3. The following first 20 observations below were drawn from a normal distribution with mean $\mu = 0$ and $\sigma = 1$. The last 20 observations were drawn from a normal distribution with mean $\mu = 1$ and standard deviation $\sigma = 1$.

The target is $T = 0$, and let's assume that it is important to detect a shift of 1 unit, so that $\delta = 1$ and $k = 0.5$, since $k = (\delta/2)\cdot\sigma = (1/2) \cdot 1 = ½$.

For single observations (samples of size 1), we have:

$C_i^- = \min\{0, x_i - (T - k) + C_{i-1}^-\} = \min\{0, x_i + 0.5 + C_{i-1}^-\}$

$C_i^+ = \max\{0, x_i - (T + k) + C_{i-1}^+\} = \max\{0, x_i - 0.5 + C_{i-1}^+\}$

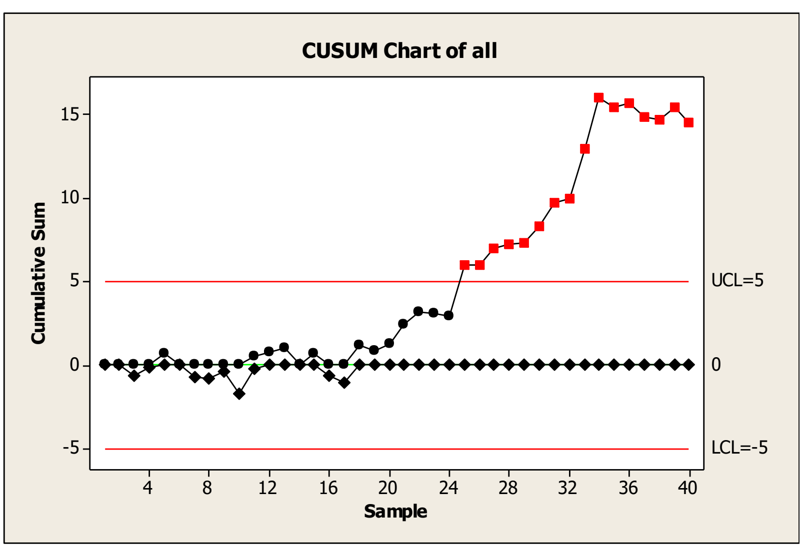
Is sum *above* 0.5? Is sum *below* -0.5?

Sample No.	x_i	$x_i - 0.5$	$(x_i - 0.5) + C_{i-1}^+$	C_i^+	$x_i + 0.5$	$(x_i + 0.5) + C_{i-1}^-$	C_i^-
1	0.31	-0.19	-0.19	0	0.81	0.81	0
2	-0.20	-0.7	-0.7	0	0.3	0.3	0
3	-1.16	-1.66	-1.66	0	-0.66	-0.66	-0.66
4	0.02	-0.48	-0.48	0	0.52	-0.14	-0.14
5	1.17	0.67	0.67	0.67	1.67	1.53	0
6	-0.14	-0.64	0.03	0.03	0.36	0.36	0
…							
20	0.93	0.43	*	1.30	1.43	1.43	0
21	1.64	1.14	2.44	2.44	2.14	2.14	0
22	1.23	0.73	3.17	3.17	1.73	1.73	0
23	0.38	-0.12	3.05	3.05	0.12	0.12	0
24	0.40	-0.10	2.96	2.96	…	…	0
25	3.54	3.04	6.00	6.00			0
26	0.47	…	…	5.97			0

C_i^+ = "cumulative sum of how much OVER 0.5"
C_i^- = "cumulative sum of how much UNDER -0.5"

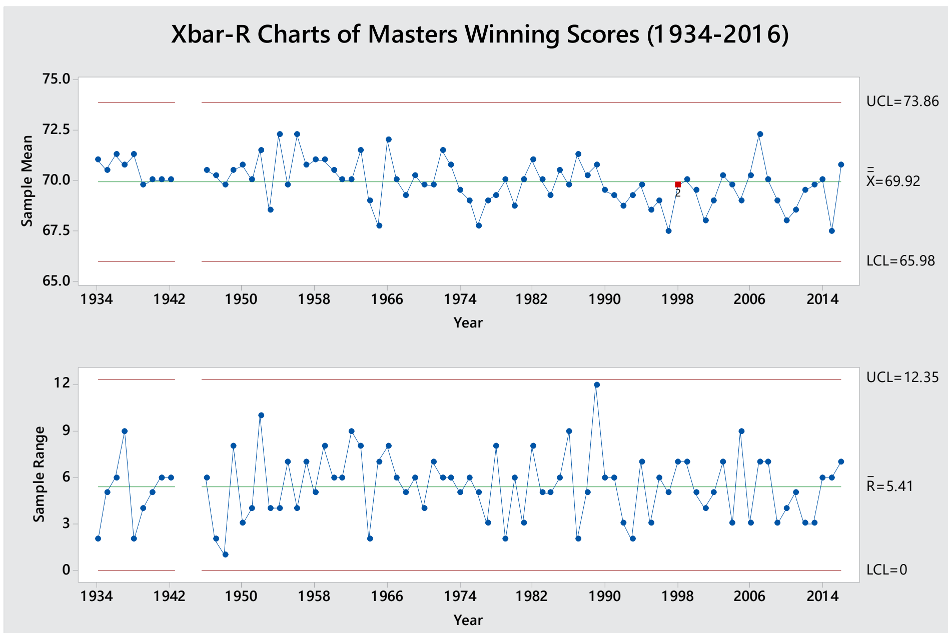
The decision interval is $h = 5 \cdot \sigma = 5 \cdot 1.0 = 5$ and $C^+ > 5$ at observation 25.

So, there is evidence that the process is out of control at sample 25 and C^+ has been positive from observation 18 on, indicating that the process may have gone out of control around observation 18 or so.

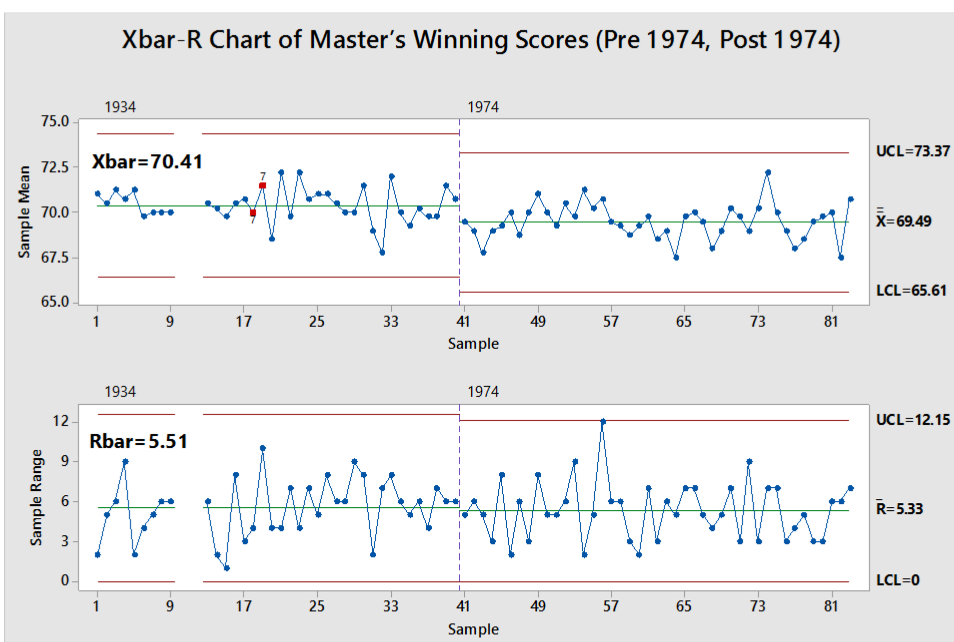


Example 4: The Masters Golf Tournament is said to be "a tradition unlike any other." The tournament that began in 1934 is held at the same course every year. Augusta National is a par 72 course. The tournament was not played in 1943, 1944, and 1945 due to World War II. I constructed a Minitab worksheet, called **Lesson21DATA_MastersTournamentScores**, with each year's winner's scores from 1934 to 2016 for the four days of the tournament.

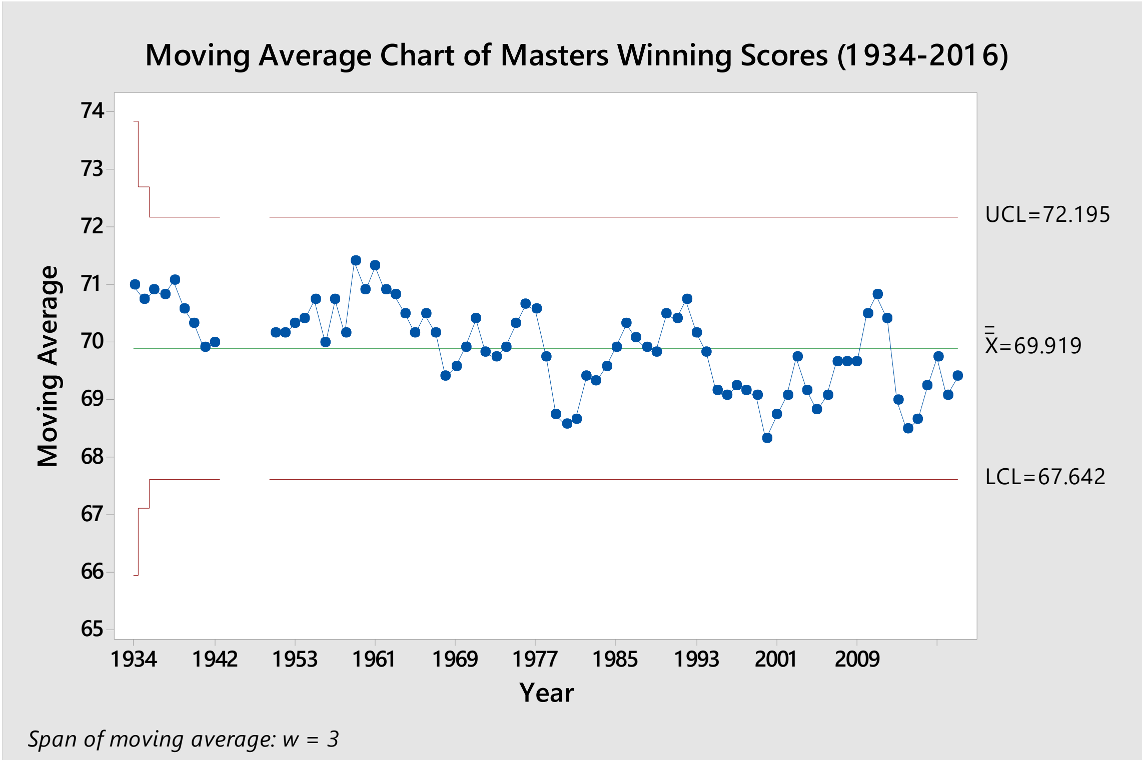
(a) Construct Xbar-R charts using the player's scores for the four rounds. Make sure the plots are appropriately labeled with a descriptive title, such as "Xbar-R Charts of Masters' Winning Scores (1934-2016)."

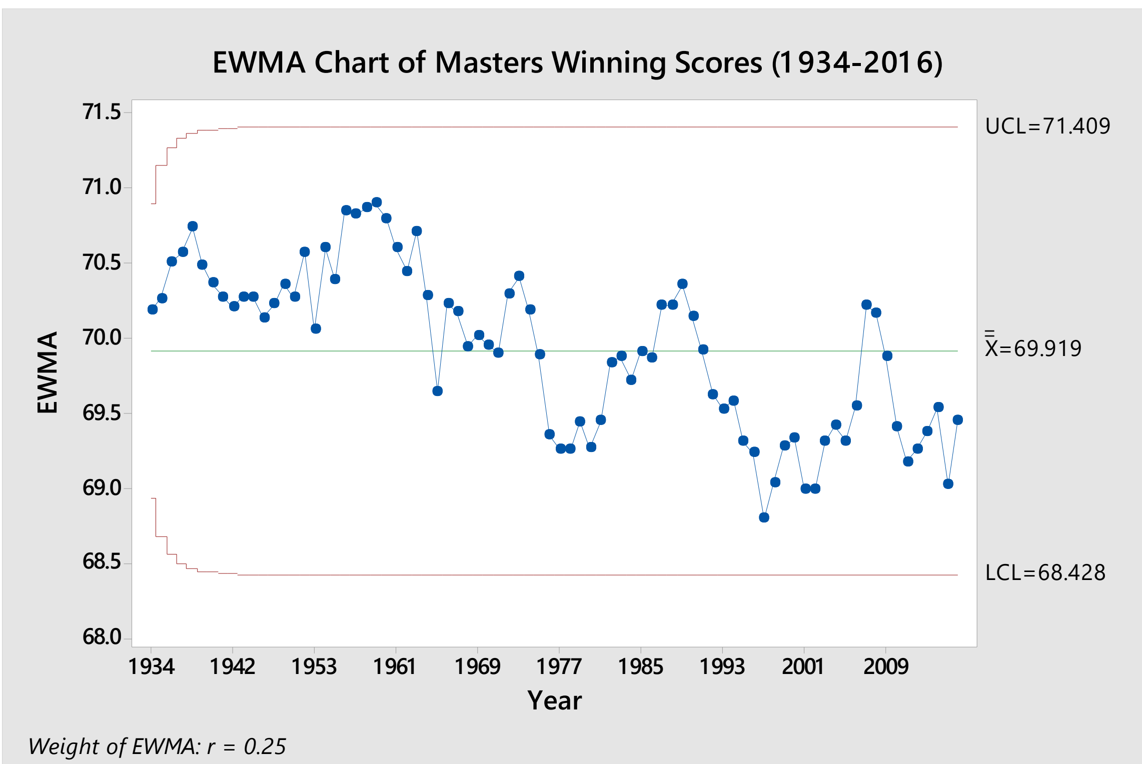


(b) The following control charts look at the data as two different processes based on pre and post 1974.



Using Time Weighted to determine a shift in the mean





CUSUM, requires parameters *T* (target), *h*, and *k*

Process standard deviation: $\hat{\sigma} = \frac{5.41}{2.059} \cong 2.63$ (from Xbar-R chart)

Target: 70.41 (overall mean)

$k = (\delta/2)\cdot\sigma$ where δ is the shift in the mean that you want to detect and σ is the process standard deviation

$$k \cong \frac{0.5}{2} \cdot 2.63 \cong 0.67$$

h is typically 4-5 times the process standard deviation; let $h = 8$